Mathbanks

VCE Mathematical Methods Unit 1/2 | Topic Test

Applying Polynomial Functions

Practice Test (SAC) | CAS

Question and Answer Book

Mathbanks Practice Assessment

- Reading time is **5 minutes**
- Writing time is **60 minutes**

Materials supplied

- Question and Answer Book of 13 pages
- Formula Sheet

Instructions

Students are permitted to bring into the assessment room: one approved CAS technology (calculator or software) and, if desired, one scientific calculator, and one bound reference.

Students are **not** permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Contents pages

Instructions

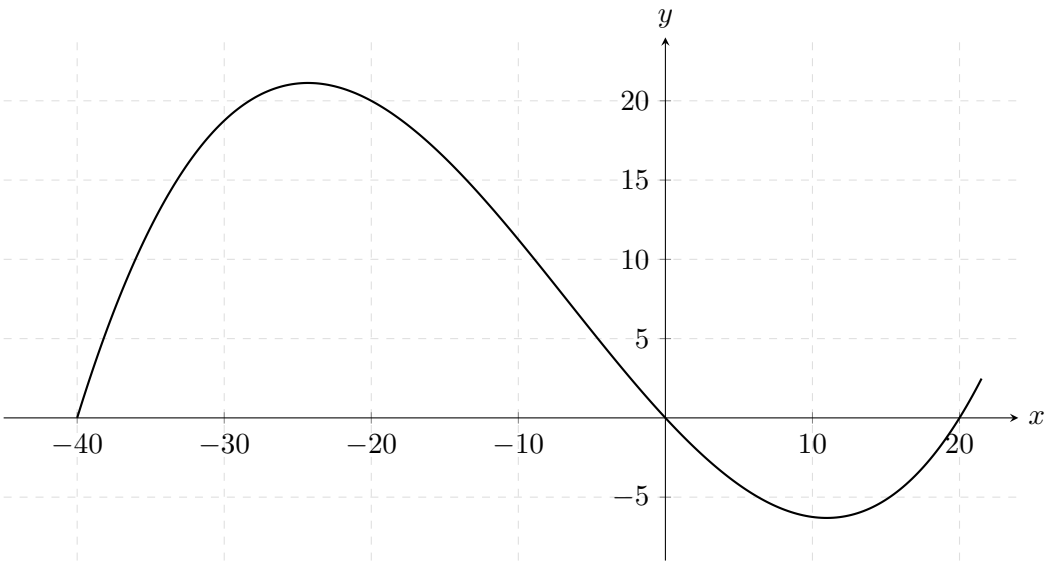
- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (11 marks)

A roller coaster has a section modelled by the curve

$$y = \frac{1}{800}\left(x^3 + 20x^2 - 800x\right),$$

where y is the height in metres above the ground and x is the distance in metres from the entrance to the tunnel (the origin). It rises from ground level at point A to a peak at B, enters a tunnel at C (ground level), reaches its lowest level at D, and exits the tunnel again at E (back to ground level). Point C is the origin $(0, 0)$.



a. Label points A, B, C, D and E on the graph. 1 mark

b. Determine the coordinates of A and E, and verify that point C is $(0, 0)$. 3 marks

c. Using calculus, find the exact distance from the entrance of the tunnel at which the roller coaster reaches its maximum height above ground level. 2 marks

d. Determine the greatest depth of the tunnel. Express your answer to two decimal places. 2 marks

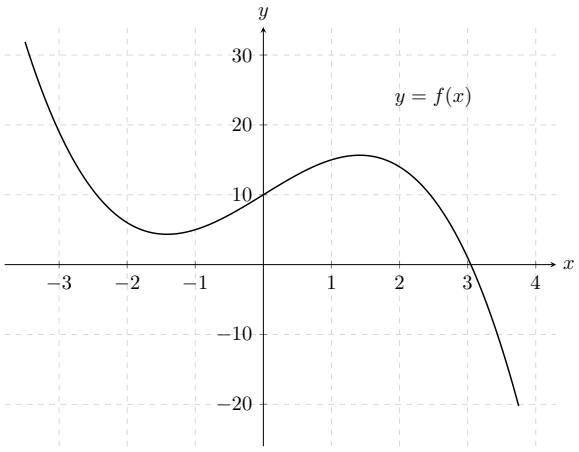
e. Determine the exact gradient of the track at the beginning and at the end of the tunnel. 1 mark

A reinforced steel beam will be a tangent to the curve along the upslope of the tunnel leading to the exit, situated at a gradient of 1.25.

f. Give the equation of the line that represents where the beam touches the wall of the tunnel (all values to two decimal places). 2 marks

Question 2 (17 marks)

The graph of the function f, where $f : \mathbb{R} \to \mathbb{R}$, $f(x) = -x^3 + 6x + 10$, is shown below.



a. Find the coordinates of the y-intercept of the graph. 1 mark

b. One of the stationary points of f is located at $\left(-\sqrt{2},\ 10 - 4\sqrt{2}\right)$. Show that the other stationary point is located at $\left(\sqrt{2},\ 10 + 4\sqrt{2}\right)$. 2 marks

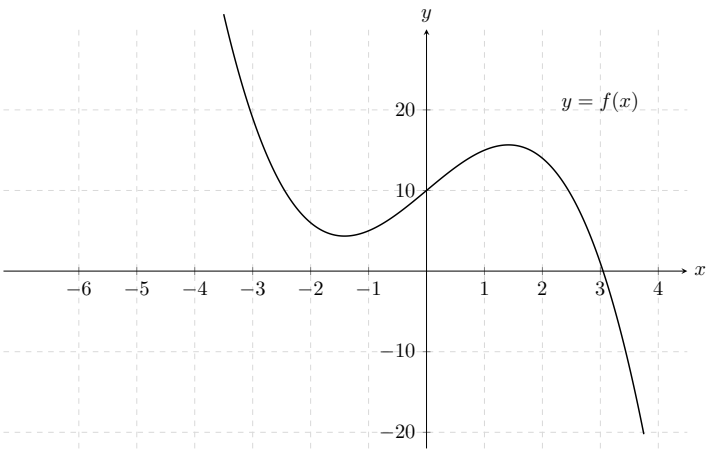
c. Find the values of x for which $f'(x) < 0$. 1 mark

d. Use Newton's method, with an initial estimate of $x_0 = 3$, applying the method twice (that is, finding x_2), to approximate the x-intercept of the graph of f. Give your answer correct to three decimal places. 2 marks

The graph of f undergoes a transformation to become the graph of g. The graph of f is

- reflected in the y-axis;
- dilated by a factor of 2 from the y-axis.

e. Sketch the graph of g on the set of axes below, where the graph of f is shown. Indicate the exact coordinates of the stationary points and the coordinates of the x- and y-intercepts, expressing the x-coordinate of the x-intercept correct to two decimal places. 4 marks



f. Find the rule for the function g. 3 marks

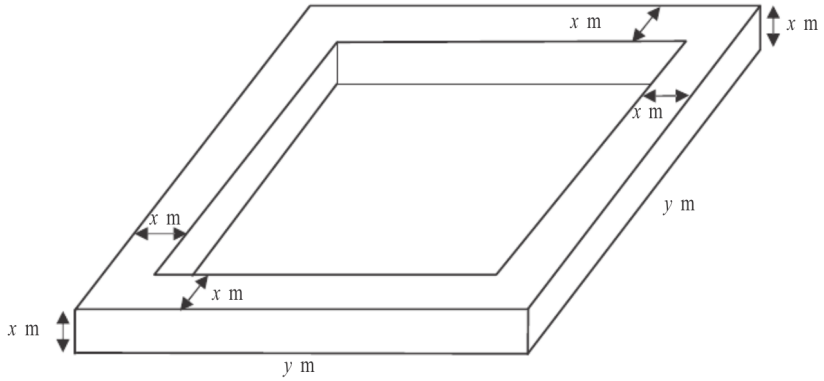
g. Find the distance between the point on the graph of f where $x = 4$ and the point on the graph of g where $x = 0$. Give your answer as an exact value. 2 marks

The function h, given by $h : [a, \infty) \to \mathbb{R}$, $h(x) = -x^3 + 6x + 10$, has the same rule as f but a different domain. The function h has an inverse function h^{-1}.

h. Find the least value that a can have. Give your answer as an exact value. 1 mark

Question 3 (12 marks)

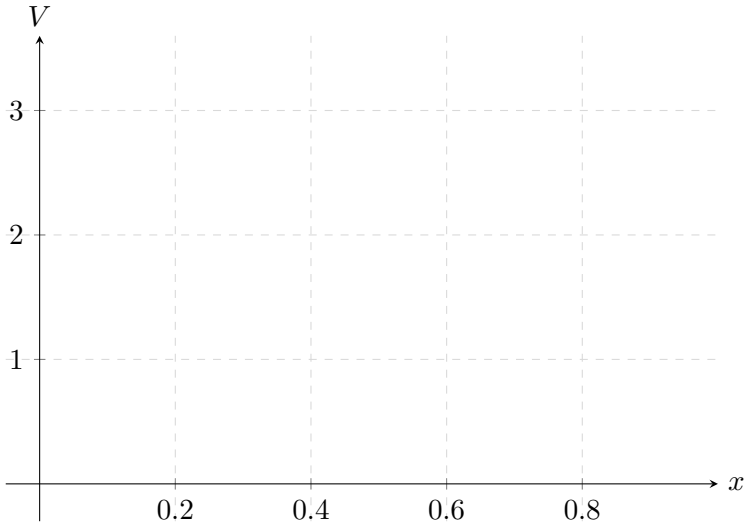
A square brick structure (a square frame) with external side length y m is to be built. The height, width and depth of each of the four side walls is x m, where $x \leq 0.9$. The external side length y and twice the wall length x must add to give a total of 3.6 m.



a. Write down an expression for y in terms of x. 1 mark

b. Show that the total volume V, in m^3, of this brick structure is given by $V = -12x^2(x - 1.2)$. 3 marks

c. On the axes below, sketch the graph of the volume function $V(x)$, showing clearly the coordinates of the endpoints. 3 marks



d. What is the maximum volume of the brick structure? Express your answer correct to one decimal place. 1 mark

e. What are the values of x and y when the volume is a maximum? 2 marks

f. If the volume of materials is limited to 2 m^3, write down a set of values that x and y could take so that $V = 2$ m^3. Express the values correct to one decimal place. 2 marks

Mathematical Methods Units 1 & 2 formulas

Functions and graphs

distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
midpoint formula	$\text{midpoint} = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$

Straight line graphs

general equation	$y = mx + c$
equation through point (x_1, y_1)	$y - y_1 = m(x - x_1)$
gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$

Mensuration

circumference of a circle	$2\pi r$	area of a circle	πr^2
volume of a sphere	$\frac{4}{3}\pi r^3$	volume of a cylinder	$\pi r^2 h$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	

About Mathbanks

Mathbanks is an online question bank for VCE Mathematics. It contains thousands of original, exam-style questions — each written with a complete worked solution and mark scheme — organised by topic and by difficulty across Mathematical Methods Units 1 & 2 and Units 3 & 4.

About This Test

This practice test features extended, application-based polynomial questions designed for CAS-enabled assessment in VCE Mathematical Methods Units 1 & 2. Students use CAS technology to model and analyse real-world and geometric situations with cubic and quartic functions, set up and solve polynomial equations, and interpret their results in context. The test develops fluency with the calculus of polynomials — finding and classifying stationary points, locating maximum and minimum values, evaluating gradients and tangents, and determining where a function is increasing or decreasing. Students construct polynomial models from given conditions, solve for unknown parameters, apply numerical techniques such as Newton's method, and interpret the effect of transformations on polynomial graphs. Application topics include volume and area optimisation, curve and path modelling, and problems where a polynomial rule must be built, solved, and interpreted with the aid of technology.

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