Mathbanks

VCE Mathematical Methods Unit 1/2 | Topic Test

Applying Exponentials & Logarithms

Practice Test (SAC) | CAS

Question and Answer Book

Mathbanks Practice Assessment

- Reading time is **5 minutes**
- Writing time is **65 minutes**

Materials supplied

- Question and Answer Book of 12 pages
- Formula Sheet

Instructions

Students are permitted to bring into the assessment room: one approved CAS technology (calculator or software) and, if desired, one scientific calculator, and one bound reference.

Students are **not** permitted to bring mobile phones and/or any other unauthorised electronic devices into the assessment room.

Contents pages

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

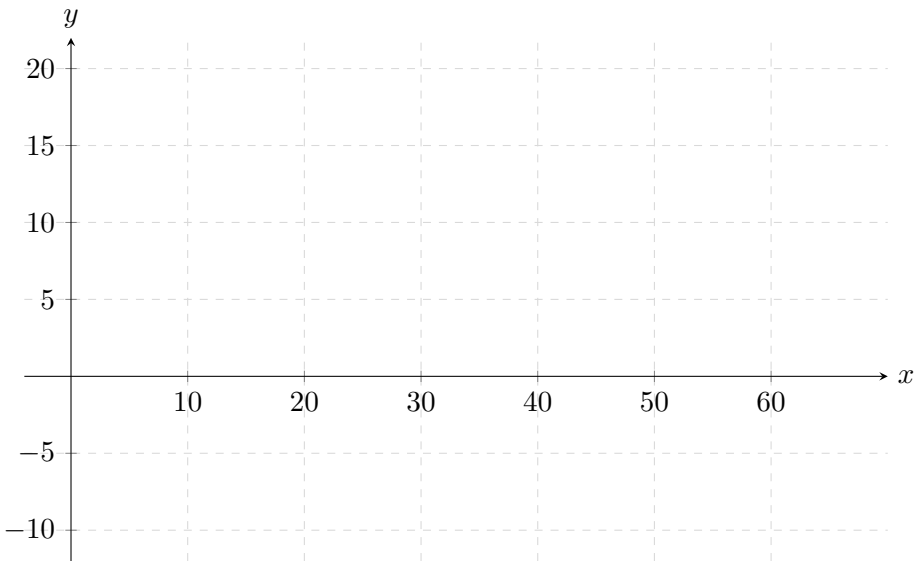
Question 1 (12 marks)

Consider the function $f : (2, 66] \to \mathbb{R},\ f(x) = 3\log_2(x-2) + 1$.

a. State the range of f. 1 mark

b. State the value of $f(9)$ correct to two decimal places. 1 mark

c. Sketch the graph of $y = f(x)$ on the axes below, labelling all key features. Label the x-intercept correct to two decimal places. 3 marks



d. Find the average rate of change between $x = 6$ and $x = 66$. 2 marks

e. Find the rule and domain of $f^{-1}(x)$, the inverse of $f(x)$. 3 marks

f. Find the coordinates of the points of intersection of $f(x)$ and $f^{-1}(x)$. Give your answer correct to two decimal places where appropriate. 2 marks

Question 2 (14 marks)

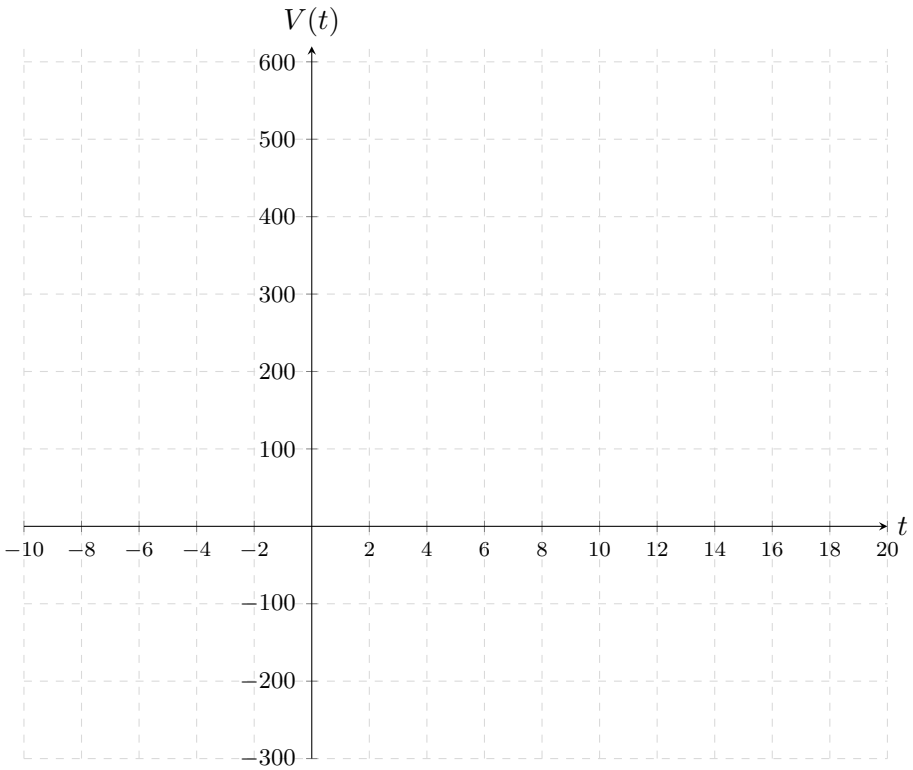
A distillery operates between the hours of 6:00 am and 8:00 pm every day, including weekends. The distillery's equipment extracts lavender oil from lavender flowers. During winter, the volume of lavender oil, in litres, that is extracted each day is given by the function

$$V(t) = 12 \times 3^{0.25t} + 6,$$

where t is the number of hours that the equipment operates per day and $t \in [0, 14]$.
The extracted lavender oil is collected in a single vessel that is emptied at the end of each day. However, the vessel cannot be emptied completely, so a small amount of oil always remains in the vessel.

a. Show that 18 L of lavender oil remains in the vessel at the start of each day during winter. 1 mark

b. Sketch the graph of $y = V(t)$ on the axes below, showing the coordinates of the endpoints. 3 marks



During spring, the distillery has the same operating hours but extracts a greater volume of lavender oil. The volume of lavender oil, in litres, extracted each day during spring is given by the function

$$V(t) = 2 \times 3^{0.02kt} + 60, \qquad t \in [0, 14].$$

c. If $V(3) = 67.4498$ L, find the value of k, correct to two decimal places. 2 marks

d. The lavender oil vessel has a capacity of 1100 L. What is the value of k such that the vessel is completely full at the end of each day during spring? Give your answer correct to two decimal places. 2 marks

Question 3 (17 marks)

The growth of a culture of a fungus is modelled by the equation

$$N = 6500 \times 3^{0.2t}, \qquad t \in [0, \infty)$$

where N represents the number of fungal cells present at time t hours.

a. State the initial number of fungal cells present. 1 mark

b. The number of fungal cells present increases by a constant factor every 5 hours. Find the value of this factor. 1 mark

c. Determine the time it will take for the number of fungal cells in the culture to reach 1 000 000. Give your answer correct to the nearest hour. 2 marks

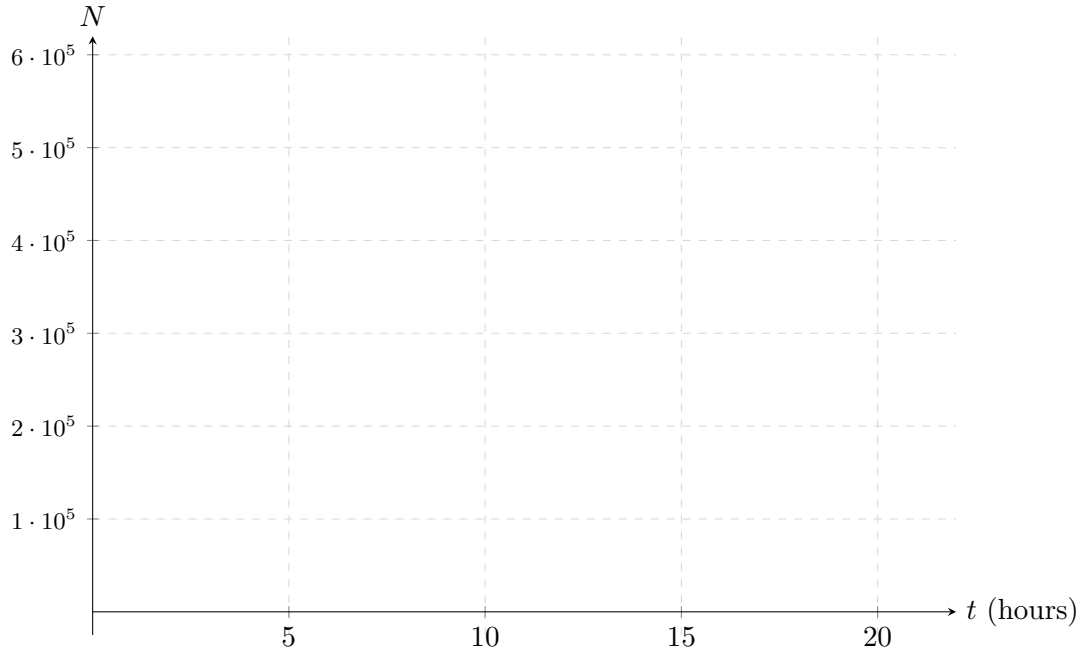
If a person is infected by the fungus, an antifungal medication is recommended. After the first dose of antifungal is administered, the quantity present in the person's body is modelled by the equation

$$N_A = 70200t - 877500, \qquad t \in [12.5, \infty)$$

where N_A represents the quantity of antifungal present at time t hours after the first dose is administered. The first dose is administered when the graphs of N and N_A first intersect. When the graphs intersect for the second time, the antifungal is no longer effective.

d. Determine the times at which the two intersections occur and the number of fungal cells that are present at each intersection. 3 marks

e. On the axes below, sketch the graphs of N and N_A. Label all endpoints and points of intersection with their coordinates. 5 marks



f. If a person who is immunocompromised is infected by the fungus, they will start to feel unwell very quickly. A different antifungal is administered, which reduces the number of fungal cells present in the person's body according to the equation

$$N_B = -5(t-5)^2 + 1750$$

where N_B is the number of fungal cells present at time t hours after the antifungal dose is administered. The antifungal is administered as soon as infection is suspected.

i. Determine the number of fungal cells present 4 hours after the antifungal is administered. 1 mark

ii. Find another time at which the number of fungal cells present is the same as at 4 hours. 1 mark

iii. Determine the time at which the number of fungal cells peaks after the antifungal is administered and the number of fungal cells present at this time. 2 marks

iv. Find the time at which the fungal cells are eliminated after the antifungal is administered. Give your answer correct to the nearest hour. 1 mark

Mathematical Methods Units 1 & 2 formulas

Functions and graphs

distance formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
midpoint formula	$\text{midpoint} = \left(\dfrac{x_1 + x_2}{2},\ \dfrac{y_1 + y_2}{2}\right)$

Straight line graphs

general equation	$y = mx + c$
equation through point (x_1, y_1)	$y - y_1 = m(x - x_1)$
gradient	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Mensuration

circumference of a circle	$2\pi r$	area of a circle	πr^2
volume of a sphere	$\dfrac{4}{3}\pi r^3$	volume of a cylinder	$\pi r^2 h$

Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$	

About Mathbanks

Mathbanks is an online question bank for VCE Mathematics. It contains thousands of original, exam-style questions — each written with a complete worked solution and mark scheme — organised by topic and by difficulty across Mathematical Methods Units 1 & 2 and Units 3 & 4.

About This Test

This practice test features application-based exponential and logarithmic questions designed for CAS-enabled assessment in VCE Mathematical Methods Units 1 & 2. Students use CAS technology to model real-world situations involving exponential growth and decay — including population and cell growth and quantities that approach a limiting value — and interpret their results in context. The test develops fluency with constructing exponential and logarithmic models from given data, solving for unknown parameters and rate constants, and using logarithms to find the time at which a modelled quantity reaches a target value. Students sketch and interpret the graphs of exponential and logarithmic functions, identify asymptotes, intercepts and endpoints, state domain and range, and locate points of intersection between models. Application topics include growth and decay, limiting behaviour, and problems where an exponential or logarithmic rule must be built, solved, and interpreted with the aid of technology.

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